Name: _____ TF: _____

Harvard University
Computer Science 121
Professor Harry Lewis

Final Examination
December 20, 2010

You should answer the Problem 1 directly on this sheet. Please write your name on the exam and return it with the blue book. Do the rest of the problems in the blue book. The points total 150.

CLIQUE $= \{\langle G, k\rangle : G$ is an undirected graph, $k > 0$, and there are some k nodes of G of which every pair is connected by an edge of $G\}$. $\Sigma = \{a, b\}$, D is a DFA, and M is a Turing machine throughout.

PROBLEM 1 (24 points)

Fill the blank entries of the following table with `YES`, `NO`, or `??` ("currently unknown"). No explanations needed.

Language:	regular	context-free	recursive	r.e.	P	NP
$\{a^i b^j c^k : i \cdot j = k\}$						
$\Sigma^* - a^* b^*$						
CLIQUE						
$\{\langle D_1, D_2\rangle : L(D_1) = L(D_2)\}$						

PROBLEM 2 (21 points)

Which of the following classes of languages are closed under union? In each case, give a counterexample and justify it, or explain why the class is closed.
(A) The class containing all languages that are either finite or co-finite.
(B) The class of languages that are context-free but not regular.
(C) The class of recursive languages.

PROBLEM 3 (15 points)

Draw a DFA that accepts all and only strings ending in *aba*.

PROBLEM 4 (10 points)

Draw a diagram showing the relations among these language classes: recursive, r.e., co-r.e., P, NP, NP-complete, co-NP,
(A) on the assumption that P = NP;
(B) on the assumption that P $\neq$ NP

(TURN OVER!)

PROBLEM 5 (5+10 points)

(A) Is $aabb \in \{ww^R : w \in \Sigma^*\}^*$?
(B) Write a context-free grammar that generates this language.

PROBLEM 6 (20 points)

Which of the following languages are recursive? Why or why not?
(A) $\{\langle M, w, k\rangle : M$ uses more than k tape squares when computing on input $w\}$
(B) $\{\langle M\rangle : L(M)$ is recursive$\}$
(C) $\{\langle M\rangle : L(M)$ is countable$\}$
(D) $\{\langle M\rangle : L(M) = \{ww : w \in \Sigma^*\}\}$

PROBLEM 7 (20 points)

Let G be an undirected graph and let k be a positive integer. Let Q be the set of all pairs $\langle G, k\rangle$ such that there is a set of k nodes of G, no two of which are connected by an edge of G. Show that Q is NP-complete. Hint: Reduce CLIQUE to this problem.

PROBLEM 8 (10+5+10 points)

(A) Show that fhere is a polynomial-time algorithm A_2 for determining whether a Boolean formula has a satisfying truth assignment in which exactly 2 of the Boolean variables are true.
(B) Show that there is a similar polynomial-time algorithm A_k for any $k \geq 0$, which determines whether a formula can be made true by making exactly k of its variables true.
(C) Then why isn't the following procedure a polynomial-time algorithm for SAT? "Given a formula F with n variables, sequentially apply A_0, A_1, ..., A_n to F and accept if one of the A_i accepts."