Name: _____ TF: _____

**Harvard University**
**Computer Science 121**
**Professor Harry Lewis**

Final Examination
December 20, 2011

**Answer Problem 1 directly on this sheet. Write your name on the exam and return it with the booklet. Do the rest of the problems in the booklet.** The points total 140.

$\Sigma = \{a, b\}$, $D$ is a DFA and $M$ is a Turing machine throughout. QBE is the set of all quantified boolean expressions, for example $\forall x \exists y \forall z[(x \wedge y) \vee (\neg x \wedge \neg z)]$.

PROBLEM 1 (25 points)

Fill the blank entries of the following table with `YES`, `NO`, or `??` ("currently unknown"). No explanations needed.

| Language: | finite | regular | context-free | recursive | r.e. | $\mathcal{P}$ | $\mathcal{NP}$ |
|---|---|---|---|---|---|---|---|
| $\{ww^{\mathrm{R}} : w \in \Sigma^*\}^*$ | | | | | | | |
| $L(a^*b^*) \cap L(b^*a^*)$ | | | | | | | |
| $\{\langle D_1, D_2\rangle : L(D_1) \cup L(D_2) = \Sigma^*\}$ | | X | X | | | | |
| True members of QBE | | | X | | | | |

PROBLEM 2 (40 points)

Answer each question briefly, by arguing the positive or giving a counterexample.
(A) The complement of a context-free language is necessarily recursive.
(B) There is a bijection between the class of co-finite languages and the class of co-r.e. languages.
(C) It is possible to determine algorithmically whether a DFA accepts only finitely many strings.
(D) It is currently unknown whether every $\mathcal{NP}$ language is recursive.
(E) The set difference between two languages in $\mathcal{P}$ is also in $\mathcal{P}$.

PROBLEM 3 (10 points)

Draw a DFA that accepts a string if and only if each occurrence of the substring $aa$ is followed immediately by the substring $bb$.

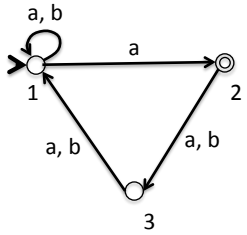
PROBLEM 4 (10 points)

Draw a diagram showing the relations among these language classes: recursive, r.e., co-r.e., $\mathcal{P}$, $\mathcal{NP}$, $\mathcal{NP}$-complete, co-$\mathcal{NP}$,
(A) on the assumption that $\mathcal{P} = \mathcal{NP}$;
(B) on the assumption that $\mathcal{P} \neq \mathcal{NP}$

(TURN OVER!)

PROBLEM 5 (20 points)

Convert this NFA to a DFA using the subset construction. Show your work.




PROBLEM 6 (30 points)

(A) Show that $\{\langle M, w, 1^k\rangle : M \text{ halts on input } w \text{ in at most } k \text{ time steps}\}$ is in $\mathcal{P}$.
(B) Show that $\{\langle M\rangle : L(M) \text{ is recursive}\}$ is not recursive.
(C) Show that there is no algorithm to determine whether a Turing machine accepts finitely many inputs, but there is an algorithm to determine whether a Turing machine accepts countably many imputs.

PROBLEM 7 (10 points)

(A) Fill in the ellipsis (. . . ), defining any terms other than "graph," "node," and "edge":

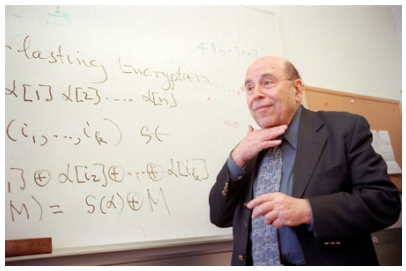
$$\textsc{Vertex Cover} = \{\langle G, k\rangle : \ldots\}$$

(B) We proved in class that Vertex Cover is $\mathcal{NP}$-complete. Does it matter whether $k$ is represented in unary or binary? Why or why not?

PROBLEM 8 (5 points)

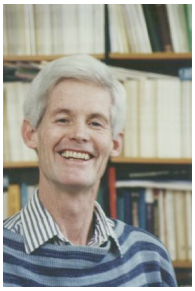
Name these CS121 heroes:

(A) 

(B) 

(C) 

(D) 

(E) 

(F) 

THE END