Harvard University
Computer Science 121

Problem Set 0

Due Tuesday, September 11, 2012 at 11:59 PM.
Submit your solutions electronically on the course website, located at
http://people.seas.harvard.edu/~salil/cs121/fall12/. On the site, click the "Problem Set Submission" button and provide your login info. Once logged in, place the solutions to Parts A and B, in seperate files named lastname+ps0a.pdf and lastname+ps0b.pdf respectively, in the appropriate dropboxes.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)**

See syllabus for collaboration policy.

Note: Problem set 0 will not count towards your final course grade, but it is strongly recommended that you complete it to assess and strengthen your mathematical preparation for the course. It will be marked and a solution will be provided.

PART A (Graded by Nick)

PROBLEM 1 (1+1+1+1+1+1 points)

Let $A = \{1, 2, \cdots, 10\}$ and $B = \{a, b, c, d, 7, 8, 9\}$.
(A) What is $A \cap B$?
(B) What is $|A \cup B|$?
(C) Give three examples of elements of $A \times B$.
(D) What is $|A \times B|$?
(E) Give three examples of elements of $P(A)$.
(F) What is $|P(A)|$?

PROBLEM 2 (2+2+2+2 points)

Let $\mathbb{N} = \{0, 1, 2, \ldots\}$ be the set of natural numbers. For each of the following functions $f : \mathbb{N} \to \mathbb{N}$, state whether f is (i) one-to-one, (ii) onto, (iii) bijective. Briefly justify your answers.
(A) $f(x) = x \mod 2$
(B) $f(x) = \lfloor e^x \rfloor$, where $\lfloor y \rfloor$ denotes the largest integer less than or equal to y.
(C) $f(x) = \begin{cases} x+1 & \text{if } x \text{ is even} \\ x-1 & \text{if } x \text{ is odd} \end{cases}$

PROBLEM 3 (3+4 points)

For a string, x, let x^R be the reverse of x (*e.g.*, $(abc)^R = cba$) and let x^i be the concatenation of i copies of x (*e.g.*, $(abc)^2 = abcabc$).

(A) Provide an inductive definition of x^i.
(B) Prove by induction that $(x^R)^i = (x^i)^R$. (Hint: Use the fact that $(xy)^R = y^R x^R$.)

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PART B (Graded by Bo)

PROBLEM 1 (7 points)

Joe the painter has 2013 cans of paint. Show that at least one of the following statements is true about Joe's paint collection:
(i) Among the cans, there are at least 5 with the same color.
(ii) Among the cans, there are at least 504 different colors of paint.

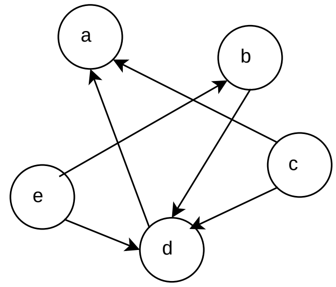
PROBLEM 2 (2+4+3 points)

Let V be the set of vertices and E be the set of edges in the directed graph G below. Define a vertex $v \in V$ to be *reachable* from a vertex $u \in V$ if $u = v$ or there is a directed path from u to v; *i.e.*, there is a sequence of edges $(x_1, x_2), (x_2, x_3), \ldots, (x_{k-1}, x_k)$ in E with $x_1 = u$ and $x_k = v$.

(A) What is the set of all vertices reachable from c?

(B) Show that *reachability* is not an equivalence relation on the graph G.

(C) What is a set of edges you can add to make *reachability* an equivalence relation on G?



PROBLEM 3 (Challenge! 2 points)

Define the Fibonacci numbers recursively as follows, where F_i indicates the i^{th} number:

$$F_0 = 0,\ F_1 = 1,\ F_n = F_{n-1} + F_{n-2}$$

Show that for $n \geq 3$, F_n is equal to the number of strings over alphabet $\{a, b\}$ of length $n - 2$ with no two consecutive b's.