Harvard University
Computer Science 121

Problem Set 0

Due Tuesday, September 11, 2012 at 11:59 PM.
Submit your solutions electronically on the course website, located at
`http://people.seas.harvard.edu/~salil/cs121/fall12/`. On the site, click the "Problem Set Submission" button and provide your login info. Once logged in, place the solutions to Parts A and B, in seperate files named lastname+ps0a.pdf and lastname+ps0b.pdf respectively, in the appropriate dropboxes.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

Note: Problem set 0 will not count towards your final course grade, but it is strongly recommended that you complete it to assess and strengthen your mathematical preparation for the course. It will be marked and a solution will be provided.

PART A (Graded by Nick)

PROBLEM 1 (1+1+1+1+1+1 points)

Let $A = \{1, 2, \cdots, 10\}$ and $B = \{a, b, c, d, 7, 8, 9\}$.
(A) What is $A \cap B$?
(B) What is $|A \cup B|$?
(C) Give three examples of elements of $A \times B$.
(D) What is $|A \times B|$?
(E) Give three examples of elements of $P(A)$.
(F) What is $|P(A)|$?

PROBLEM 2 (2+2+2+2 points)

Let $\mathbb{N} = \{0, 1, 2, \ldots\}$ be the set of natural numbers. For each of the following functions $f : \mathbb{N} \to \mathbb{N}$, state whether f is (i) one-to-one, (ii) onto, (iii) bijective. Briefly justify your answers.
(A) $f(x) = x \mod 2$
(B) $f(x) = \lfloor e^x \rfloor$, where $\lfloor y \rfloor$ denotes the largest integer less than or equal to y.
(C) $f(x) = \begin{cases} x+1 & \text{if } x \text{ is even} \\ x-1 & \text{if } x \text{ is odd} \end{cases}$

PROBLEM 3 (3+4 points)

For a string, x, let x^R be the reverse of x (*e.g.*, $(abc)^R = cba$) and let x^i be the concatenation of i copies of x (*e.g.*, $(abc)^2 = abcabc$).

(A) Provide an inductive definition of x^i.
(B) Prove by induction that $(x^R)^i = (x^i)^R$. (Hint: Use the fact that $(xy)^R = y^R x^R$.)