Harvard University
Computer Science 121
Problem Set 2

Due Tuesday, September 25, 2012 at 11:59 PM.
Submit your solutions electronically on the course website, located at `http://people.seas.harvard.edu/~salil/cs121/fall12/`. On the site, click the "Problem Set Submission" button and provide your login info. Once logged in, place the solutions to Parts A, B, and C in seperate files named lastname-ps2a.pdf, lastname-ps2b.pdf, and lastname-ps2c.pdf respectively, in the appropriate dropboxes.
Late problem sets may be turned in until Friday, September 28, 2012 at 11:59 PM with a 20% penalty.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

Unless specified otherwise, assume the alphabet $\Sigma = \{a, b\}$

PART A (Graded by Joe)

PROBLEM 1 (2+2+2+2+2 points)

Translate the following languages from set notation with English description to regular expressions, or vice versa:

(A) $((a \cup b)^*a) \cup ((a \cup b)^*b)$
(B) $(((a \cup b)a)^* \cup ((a \cup b)b)^*)(a \cup b)$
(C) $L = \{w \in \Sigma^* : w \text{ has no more than 2 } b\text{'s}\}$
(D) $L = \{w : w \text{ has length } 3\}$
(E) $L = \{w \in \Sigma^* : w \text{ has no consecutive b's}\}$

PROBLEM 2 (6+6 points)

(A) Prove that if R is a regular expression that contains no occurrences of *, then $L(R)$ is finite. Note: You may assume (without proof) basic facts about finite sets for this problem; however, *explicitly* state any such assumptions you make. [Hint: use structural induction on R.]
(B) Prove or disprove: every regular expression that does contain an occurrence of * generates an infinite language.

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PART B (Graded by Colin)

PROBLEM 1 (12 points)

(A) Construct a DFA for $L = \{w \in \Sigma^* :$ there are an even number of a's and an even number of b's in $w\}$

(B) Convert your DFA for L to a regular expression using the GNFA construction described in lecture and in Sipser (p. 66, Lemma 1.60 in the Second Edition). Show the steps of the construction. As you go along, use basic simplifications, such as $(a \cup \varepsilon)^* \to a^*$ to make the REs simpler.

PROBLEM 2 (1+1+1+1+1+1+1 points)

Classify the following sets as finite (in which case state the cardinality), countably infinite, or uncountably infinite. Give a sentence of justification.
(A) The set of syntactically valid Python programs
(B) $\mathbb{Q} \times \mathbb{Q}$ (where $\mathbb{Q}$ is the set of rational numbers)
(C) $\emptyset$
(D) $\{\emptyset\}$
(E) The set of finite languages over the alphabet $\{a, b\}$
(F) Letting $\mathbb{P}$ be the prime numbers, the set $\{R : R \subseteq \mathbb{P}\}$
(G) $\{(x_1, x_2, \ldots) :$ each $x_i \in \mathbb{N}$ and for all i, $x_i \leq x_{i+1}\}$

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PART C (Graded by Bo)

PROBLEM 1 (1+1+10 points)

Let Σ and Δ be alphabets. Consider a function $\psi : \Sigma \to \Delta^*$. Extend ψ to a function from $\Sigma^* \to \Delta^*$ using the inductive rules:

$$\psi(\varepsilon) = \varepsilon$$
$$\psi(w\sigma) = \psi(w)\psi(\sigma) \text{ , for any } w \in \Sigma^*, \sigma \in \Sigma$$

(A) For example, consider $\Sigma = \{a, b\}$ and $\Delta = \{a, b, c, d\}$ and let $\psi(a) = ccb$, $\psi(b) = dda$. What is $\psi(aba)$?

(B) Any function $\psi : \Sigma^* \to \Delta^*$ defined from a function $\psi : \Sigma \to \Delta^*$ in this way is termed a *homomorphism*.

Now for a language L and homomorphism ψ, define

$$\psi(L) = \{\psi(w) : w \in L\}$$

For example, we can extend ψ as defined in (A) to languages. If $L = \{bb, aa, ba\}$, then what is $\psi(L)$ in this case?

(C) Prove that the set of regular languages is closed under homomorphism. That is, if L is regular and ψ is a homomorphism, then $\psi(L)$ is regular.

PROBLEM 2 (Challenge! 2 points)

Let $L\backslash A = \{x : wx \in A \text{ for some } w \in L\}$. Show that if A is regular and L is *any* language, then $L\backslash A$ is regular.