Harvard University Extension School
Computer Science E-207

Problem Set 3

Due Friday, October 5, 2012 at 11:59 PM Eastern Time.
Submit your solutions in a single PDF called lastname+ps3.pdf emailed to cscie207@seas.harvard.edu.
LATE PROBLEM SETS WILL NOT BE ACCEPTED.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

Unless specified otherwise, assume the alphabet $\Sigma = \{a, b\}$

PROBLEM 1 (4+4+4+4+4 points)

For each of the following languages, determine whether the language is regular or non-regular, and *prove* your answer.

(A) $\{a^n b a^n : n \geq 1\}$
(B) $\{a^{3n} b^{5m} : n, m \in \mathbb{N}\}$
(C) $\{a^n b^m : n \geq m\}$
(D) $\{a^{2^n} : n \in \mathbb{N}\}^*$
(E) $\{w \in \Sigma^* : w$ contains exactly one of the substrings 'alan', 'matheson', or 'turing'$\}$, where Σ is the English alphabet (in lower case)

PROBLEM 2 (3+3+3+3+3+3 points)

Classify the following sets as finite (in which case state the cardinality), countably infinite, or uncountably infinite. Give a *brief* justification. (A proof is not necessary.)
(A) $\{\emptyset\}$
(B) The set of syntactically valid computer programs in the programming language Python
(C) $\mathbb{Q} \times \mathbb{Q}$ (where $\mathbb{Q}$ is the set of rational numbers)
(D) $\{R \subseteq \mathbb{N} :$ all numbers in R are prime$\}$
(E) $\{(x_1, x_2, \ldots) :$ each $x_i \in \mathbb{N}$ and for all i, $x_i \leq x_{i+1}\}$
(F) $\{(x_1, x_2, \ldots) :$ each $x_i \in \mathbb{N}$ and for all i, $x_i \geq x_{i+1}\}$

PROBLEM 3 (6 points)

Show that a DFA with n states accepts an infinite language if and only if it accepts some string of length at least n.

PROBLEM 4 (6+6+2 points)

Consider the language $F = \{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

(A) Show that F is not regular.
(B) Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the following conditions: if s is any string in F of length at least p, then s may be divided into three pieces, $s = xyz$, such that

(i) for each $i \geq 0$, $xy^iz \in F$
(ii) $|y| > 0$
(iii) $|xy| \leq p$

(C) Explain why the results from parts (A) and (B) do not contradict the pumping lemma.

PROBLEM 5 (Challenge 2 points)

Let L be the language of prime numbers in binary. That is $L = \{10, 11, 101, 111, 1011, 1101, \cdots\}$. Prove that L is not regular. (Hint: Fermat's little theorem states that, for any prime p and positive integer x that is not a multiple of p, $x^{p-1} \bmod p = 1$.)