Harvard University
Computer Science 121

Problem Set 7

Due Tuesday, November 6, 2012 at 11:59 PM.
Submit your solutions electronically on the course website, located at `http://people.seas.harvard.edu/~salil/cs121/fall12/`. On the site, click the "Problem Set Submission" button and provide your login info. Once logged in, place the solutions to Parts A, B, and C in seperate files named lastname-ps7a.pdf, lastname-ps7b.pdf, and lastname-ps7c.pdf respectively, in the appropriate dropboxes.
Late problem sets may be turned in until Friday, November 9, 2012 at 11:59 PM with a 20% penalty.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

See syllabus for collaboration policy.

Note: For all proofs involving constructions of Turing Machines, please give a *high-level* description. For examples, see the lecture on October 30 or Sipser Chapter 4.1.

PART A (Graded by Perry)

PROBLEM 1 (10 points)

Let $L = \{\langle M \rangle \mid M$ is a DFA such that, for all $w \in \Sigma^*$, if M accepts w, then it accepts $w^{\mathrm{R}}\}$. Show that L is decidable.

PROBLEM 2 (10 points)

Let $L = \{\langle M \rangle \mid M$ moves left on the tape at some point when run on $\varepsilon\}$. Show that L is decidable.

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PART B (Graded by Bo)

PROBLEM 3 (10 points)

Show that a language L_1 is Turing-recognizable if and only if there exists some decidable language L_2 such that $L_1 = \{x : \text{ there exists } y \text{ such that } \langle x, y\rangle \in L_2\}$.

(Hint: Imagine that y gives you some information about an accepting computation on x, if one exists.)

PROBLEM 4 (8 points)

Show that a language L is decidable if and only if there is an enumerator that outputs the elements of L in lexicographic order.

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PART C (Graded by Thomas)

PROBLEM 5 (5 points)

Show that the class of Turing-recognizable languages is closed under union.

PROBLEM 6 (2+2+5+(2) points)

A *general grammar* is like a CFG except that the production rules are not restricted to transforming one variable at a time without any context information. The left hand side of each rule can contain multiple variables and alphabet symbols (rather than just one variable), but there must be at least one variable on the left hand side. That is, the rules are of the form $w \to w'$ where $w, w' \in (\Sigma \cup V)^*$ and $w \notin \Sigma^*$. For example, consider the general grammar $G = (V, \Sigma, R, S)$ with $V = \{S, B, C\}$, $\Sigma = \{a, b, c\}$, and R containing the following rules:

$S \to aSBC|\varepsilon$,

$CB \to BC$,

$aB \to ab$,

$bB \to bb$,

$bC \to bc$,

$$cC \to cc$$

This yields the string *aabbcc* as follows.

$$S \Rightarrow aSBC \Rightarrow aaSBCBC \Rightarrow aaBCBC \Rightarrow aaBBCC \Rightarrow aabBCC \Rightarrow aabbCC \Rightarrow aabbcC \Rightarrow aabbcc.$$

(A) What language does the above example general grammar generate? Is this language context-free?

(B) Give a formal definition for what it means for a general grammar G to generate a string $w \in \Sigma^*$.

(C) Show that every language generated by a general grammar is Turing-recognizable. (Hint: Use nondeterministic Turing machines.)

(D) (Challenge) Prove that every Turing-recognizable language can be generated by a general grammar. This, along with the previous part, implies that general grammars are equivalent to Turing-recognizable languages. (Hint: You may assume that, without loss of generality, a Turing machine only halts with an empty tape and the head at the beginning of the tape.)