Harvard University
Computer Science 121

Problem Set 8

Due Tuesday, November 13, 2012 at 11:59 PM.
Submit your solutions electronically on the course website, located at `http://people.seas.harvard.edu/~salil/cs121/fall12/`. On the site, click the "Problem Set Submission" button and provide your login info. Once logged in, place the solutions to Parts A, B, and C in seperate files named lastname-ps8a.pdf, lastname-ps8b.pdf, and lastname-ps8c.pdf respectively, in the appropriate dropboxes.
Late problem sets may be turned in until Friday, November 16, 2012 at 11:59 PM with a 20% penalty.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)**

See syllabus for collaboration policy.

PART A (Graded by Jiapeng)

PROBLEM 1 (5+5+5 points)

Determine whether the following languages are decidable, and justify your answers.
(A) $\{\langle M\rangle : M$ eventually writes an a to the tape when run on the empty string$\}$.
(B) $\{\langle M, x\rangle : M$ accepts x within $|x|^3$ steps$\}$.
(C) $\{\langle M\rangle : M$ accepts only even-length strings$\}$.

PROBLEM 2 (10 points)

Prove that there are two languages L_Y and L_N such that

(a) L_Y and L_N are disjoint

(b) There is a TM A that halts on all inputs in $L_Y \cup L_N$, accepts all inputs in L_Y, and rejects all inputs in L_N.

(c) There is no TM B that halts on all inputs in Σ^*, accepts all inputs in L_Y, and rejects all inputs in L_N.

(Hint: take $L_Y = \{\langle M\rangle : M$ accepts $\langle M\rangle\}$ and $L_N = \{\langle M\rangle : M$ rejects $\langle M\rangle\}$, and for (c) consider a TM C that does the opposite of B.)

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PART B (Graded by Joe)

PROBLEM 3 (20 points)

For your summer internship at the software company TeamTuring.com, you are tasked with writing a program that will automatically clean up the bloated and buggy code written by the company's engineers. Specifically, your program should take a RAM TM M as input, and:

(a) Detect if M has any unreachable code (i.e. parts of the state diagram that cannot be reached on any input to M), and remove it.

(b) Detect whether M would ever write to a certain unsafe region of memory.

(c) Modify M so that it never writes to a certain unsafe region of memory, without changing its behavior on executions that wouldn't write to the unsafe region of memory.

After 8 weeks of many sleepless nights, you finally give up on successfully completing the entire project, and you settle for just accomplishing some of the items above. Which items were you able to do, and how would you convince your mentor that the others were impossible?

PROBLEM 4 (10 points)

Show that a language L is decidable if and only if $L \leq_m \{a\}$.

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PART C (Graded by Nick)

PROBLEM 5 (20 points)

Consider the problem of determining whether a PDA P accepts some string of the form $w\$w$; that is, consider the language $\{\langle P\rangle : \text{there exists a string } w \in \Sigma^* \text{ such that } w\$w \in L(P)\}$. Show that this language is undecidable.

PROBLEM 6 (Challenge!! 1 points)

Let $L = \{\langle M, x\rangle : M \text{ moves to the left no more than 121 times on input x}\}$. Determine, with proof, whether L is decidable.