Harvard University Extension School
Computer Science E-207

Problem Set 9

Due Monday, November 26, 2012 at 11:59 PM Eastern Time.
Submit your solutions in a single PDF called lastname+ps9.pdf emailed to cscie207@seas.harvard.edu.
LATE PROBLEM SETS WILL NOT BE ACCEPTED.

Problem set by **ENTER YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)**

See syllabus for collaboration policy.

PROBLEM 1 (2+2+2+2+2+2 points)

For each of the following, state whether it is true or false, and prove your assertion. For parts A-C, if the statement is true, you should state and justify values for the actual parameters n_0 and c.

(A) $4n^3 + 2n^2 = O(n^3 + n)$.
(B) $\log_2 n = \Theta(\log_{10}(n^2))$.
(C) $2^{n/2} = \Omega(2^n)$.
(D) $1.00001^n = \omega(n^{100001})$.
(E) $(2n + 1)^3 = n^3 + O(n^2)$.
(F) $n^{o(1)} = o(n)$.

PROBLEM 2 (2+2+2+2 points)

Let $f(n)$ and $g(n)$ be functions from $\mathbb{N} \to \mathbb{R}^+$. Prove or disprove the following statements.

(A) $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.
(B) $f(n) = \Theta(g(n))$ implies $g(n) = \Theta(f(n))$.
(C) $f(f(n)) = \omega(f(n))$ for all strictly increasing f (i.e. $f(n + 1) > f(n)$ for all n).
(D) $f(n) + o(f(n)) = \Theta(f(n))$.

PROBLEM 3 (3+7 points)

Prove that the class **P** is closed under:
(A) Concatenation.
(B) Kleene star. (*Hint:* Look at the algorithm we gave in class for recognizing context-free languages via Chomsky Normal Form.)

PROBLEM 4 (5+5 points)

In the following, we represent an integer polynomial

$$p(x) = \sum_{i=0}^{\deg(p)} a_i x^i$$

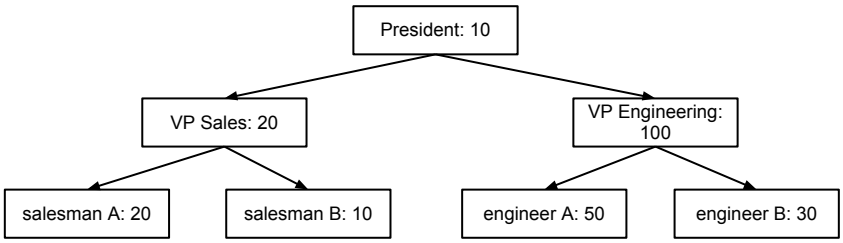
by its coefficients $a_0, \ldots, a_{\deg(p)}$ in binary, where $\deg(p)$ is the degree of p.

(A) Show that the polynomial evaluation function $\text{Eval}(\langle p, b\rangle) = \langle p(b)\rangle$ is computable in polynomial time, where p is an integer polynomial as above and b is an integer (represented in binary).
(B) Show that the polynomial composition function $\text{Comp}(\langle p_1, p_2, \ldots, p_k, b\rangle) = \langle p_1(p_2(\ldots(p_k(b))\ldots))\rangle$ *cannot* be evaluated in polynomial time, where k is allowed to vary as part of the input.

PROBLEM 5 (3+10 points)

Suppose a business with n employees is represented by a tree T. Each node represents an employee, and an edge from node a to b represents the relation "a is an immediate supervisor of b". You are consulted as a fun czar for this business and have been given a "fun index" for everybody in the business.
You want to throw the business a fun party, but a party is no fun if an employee might meet his/her supervisor in the party. So, you have to design an invite list that includes the most fun people (that is, you want to maximize the sum of the fun indices of people at the party), but still avoids anyone having to meet his/her immediate supervisor.

(A) Suppose that the President (root of the tree) has fun index 10 and supervises the VP of Sales (having fun index 20) and the VP of Engineering (having fun index 100). The VP of Sales has two subordinates: salesman A has fun index 20, and salesman B has fun index 10. The VP of Engineering has two subordinates: engineer A has fun index 50 and engineer B has fun index 30.



Calculate the most fun party and its fun index for this example.

(B) Design a general polynomial-time algorithm to find the list of people that will make the party most fun. (If there is more than one such lists, just output one).

Your algorithm, MOST-FUN-PARTY-EVARR, should take as input $\langle T, (e_0, f_0), (e_1, f_1), \ldots (e_n, f_n)\rangle$ where T is the supervisory tree and f_i is the fun index of employee e_i (each also a node of the tree). Your algorithm should output a list of employees. (*Hint*: Starting from the leaves, solve subproblems for each subtree, both when the root of the subtree is included or not included in the party.)