CS 121 Section 3

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1 Concept Review

1.1 Cardinalities

We classify the cardinality of a set S as follows.

- Finite, if there is a bijection between S and $\{1, 2, \cdots, n\}$ for some $n \geq 0$.
- Coutably infinite, if there is a bijection between S and $\mathbb{N}$.
- Countable, if it is finite or countably infinite.
- Uncountable, otherwise.

Examples include the following.

- Finite: Σ (alphabet), states of a DFA, students in CS121, finite unions of finite sets.
- Countably infinite: Σ^* (strings), $\mathbb{Z}$, DFAs, countable unions of countably infinite sets.
- Uncountable: $\mathcal{P}(\mathbb{N})$, set of all languages.

Since there are only countably many regular languages and uncountably many languages, 'most' languages are non-regular.

1.2 Pumping Lemma

Theorem 1.1 (Pumping Lemma)**.** *For any regular language L, there exists an integer p (the* pumping length *of L), such that for any string $w \in L$ with $|w| \geq p$, we can write $w = xyz$, where $|xy| \leq p$ and $|y| > 0$ and $xy^n z \in L$ for all $n \geq 0$.*

This result is useful for proving that a language is not regular: We take a string in the language and use the pumping lemma to produce a string that must be in the language, but contradicts the requirements of the language.

2 Exercises

Exercise 2.1. *Are the following sets finite (if so, how large), countably infinite, or uncountably infinite? Justify your answer.*

1. *The set of all infinite binary sequences* $\{0,1\}^{\mathbb{N}}$
2. *The set of real numbers* $\mathbb{R}$.
3. *The set of rational numbers* $\mathbb{Q}$.
4. *The set of finite languages over the alphabet* $\{a,b\}$
5. *The set of all English words.*
6. *The set of all English sentences.*

Exercise 2.2. *Are the following languages regular or non-regular. Justify your answers.*

1. $L = \{a^n b^n : n \geq 0\}$.
2. $L = \{a^m b^n : m \neq n\}$.
3. L *such that* L *is finite.*
4. $L = L_1 \cdot L_2 \cdot \cdots \cdot L_n$, *where each* L_i *is regular.*
5. $L = \{a^p : p \text{ is prime}\}$
6. $L = \{a^{p_1+p_2} : p_1, p_2 \text{ are primes}\}$.
7. $L = \{a^r : \text{ there are } r \text{ consecutive 7s appearing in the decimal representation of } \pi\}$

Exercise 2.3. *Let* $\textsc{Fraternal-Twin}(L) = \{wy : w, y \in L \text{ and } |w| = |y|\}$.

1. *Is* $\textsc{Fraternal-Twin}$ $(L((a \cup b)^*))$ *regular?*
2. *Is* $\textsc{Fraternal-Twin}$ $(L(a^* \cup b^*))$ *regular?*
3. *Show that if* L *is a regular language over* $\Sigma = \{a\}$, *then* $\textsc{Fraternal-Twin}(L)$ *is regular.*